INSTABILITY OF A NON-SELF-MAINTAINED DISCHARGE INITIATED BY PULSED IONIZATION

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UDC 537.525

It is well known that in non-self-maintained discharge, both steady-state and pulsed, instabilities develop which lead to spark breakdown. The different mechanisms responsible for instability in a steady-state discharge have been discussed, for example, in [1-5]. In particular, in [2] the effect of multistage ionization on the time increase of the electron concentration in the discharge to which a steady source of ionization is applied is considered. The problem of the instability in a discharge induced by pulsed ionization has been considered to a much less extent [6, 7]. The purpose of the present paper is to investigate the possibility of an unstable increase in the electron concentration after switching off the ionization source within the framework of the results obtained in [2].

The initial system of equations is similar to that described in [2] and consists of the electron balance equation and the excited-particle balance equation. However, unlike [2], where the equations for the electrons are considered in the quasistationary approximation, here we take into account the derivative dn_e/dt, where n_e is the electron density. In the excited-particle balance equation, as in [2] we neglect quenching collisions and "depletion" of excited particles due to multistage ionization. Hence, we have

$$\frac{dn_e}{dt} = Q + k_2 n_e n - \beta n_e^2 - \gamma N n_e, \qquad dn/dt = k_1 N n_e, \tag{1}$$

where n is the number of excited particles; k_1 and k_2 are the rate constants of excitation of the molecule by electron collision and ionization of the excited molecules; β and γ are the recombination and capture coefficients; and Q is the number of ions produced by the sources of ionization per unit volume of the gas in unit time. If the instant when the ionizing source is switched off is taken as the origin of the time measurement, then by considering the transient after the source is switched off in (1) we must put $Q = 0$, in which case $n_e(0) \neq 0$ and $n(0) \neq 0$. In this formulation the solution of system (1) is considered separately for the capture mode ($\beta = 0$) and recombination mode ($\gamma = 0$). Since in the capture case the condition for n_e and n to increase depends on the ratio of the initial values $n_e(0)$ and $n(0)$, in order to check the realization of this relation when the source of ionization acts system (1) is considered for the capture mode and for $Q \neq 0$, in which case $n_e(0) = n(0) = 0$, since the initial instant of time coincides with the instant when the ionization source is switched on.

1. The Recombination Mode. Expanding system (1) in terms of n and reducing the order of the equation, the following relation connecting the variables n_e and n at an arbitrary instant of time can be obtained:

$$k_1 N n_e = (k_2/a)(n - 1/a) + Ce^{-an}, \tag{2}$$

where $C = (k_1 n_e(0)N - (k_2/a)\,[n(0) - 1/a])e^{an(0)}$; $a = \beta/k_1 N$. It is assumed that $k_2 n(0) < \beta n_e(0)$, and at the initial instant the electron concentration is reduced [the case when $k_2 n(0) > \beta n_e(0)$ leads to an obvious increase in n_e after the source is switched off].

We will find the condition that at a certain instant ($t = t_1$) the relation $dn_e/dt = 0$ is satisfied. Substituting $n_e = (k_2/\beta)n$ into Eq. (2), we obtain an equation for n at this point:

$$n(t_1) = \frac{1}{a} \ln \frac{a^2 C}{k_2}. \tag{3}$$

From the condition $n > 0$ we obtain

$$(\beta n_e(0) - k_2 n(0)) + k_1 k_2 N/\beta > (k_1 k_2 N/\beta)e^{-an(0)}. \tag{4}$$

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 18-22, May-June, 1978. Original article submitted May 12, 1977.

0021-8944/78/1903-0293$07.50

Taking into account the condition $k_2n(0) < \beta n_e(0)$, it is seen that inequality (4) is always satisfied. This means that within the framework of the model employed, after the source of ionization is switched off the electron concentration first falls, but then reaches a minimum and then increases without limit. In fact, for sufficiently large n, from Eq.(2) we have $k_1Nn_e \simeq (k_2/a)(n-1/a)$, in which case $k_2n > \beta n_e$, so that $dn_e/dt > 0$. Since $(k_2n-\beta n_e) \to k_1k_2N/\beta$, asymptotically, it follows from Eq. (1) that the increase in n_e occurs exponentially with constant β/k_1k_2N. As regards order of magnitude, the time t_i for the instability to develop is made up of the time t_2 taken for the electron concentration to reach the minimum and the exponential rise time. For given $n_e(0)$ and $n(0)$, the time t_2 can be

$$t_2 = \int\limits_{n(0)}^{n(t_1)} \left[\frac{k_2}{a}\left(n - \frac{1}{a}\right) + C\,e^{-an}\right]^{-1} dn.$$

We will estimate the time t_i in nitrogen as it applies to the experimental conditions in [8], in which for $E/p \sim (20\text{-}30)$ V/cm · mm Hg, values of the retardation time of spark breakdown t_r are given. Since this comparison is purely an estimate, we will assume that the time t_i in the model problem described by system (1) is determined by the characteristic time parameter β/k_1k_2N, assuming in this case that multistage ionization in nitrogen occurs with greatest probability from the metastable level $A^3\Sigma_u^+$. Using the data given in [9] for the rate constants of excitation of this level and ionization from it, we obtain, in particular, for $E/p \simeq 20$ V/cm · mm Hg, $t_i \sim \beta/k_1k_2N \sim 3 \cdot 10^{-7}$ sec (here we have put $\beta = 2 \cdot 10^{-7}$ cm^3/sec and $N = 2 \cdot 10^{20}$ cm^3), which agrees in order of magnitude with the delay time of spark breakdown given in [8], which has a value of the order of 10^{-7} sec for the same value of the field, The experimentally observed strong dependence of the time t_d on the field in this model can be explained by the sharp dependence of the product k_1k_2 on E/p.

2. The Capture Mode. We will also consider the solution of system (1) in this case when $dn_e(0)/dt < 0$. The nature of the solution depends on the sign of A, defined by the following expression:

$$A = (1/2)(k_2n(0) - \gamma N)^2 - k_1k_2Nn_e(0). \tag{5}$$

If $A > 0$, the solution for n_e has the form

$$t + B_1 = \varphi(n_e), \tag{6}$$

where

$$\varphi(n_e) = -\frac{1}{\sqrt{2A}}\ln\left\{\frac{\sqrt{A + k_1k_2\,Nn_e} - \sqrt{A}}{\sqrt{A + k_1k_2\,Nn_e} + \sqrt{A}}\right\};\quad B_1 = \varphi\,(n_e(0)).$$

It is seen from (6) that n_e approaches zero monotonically as $t \to \infty$. If $A < 0$, we have for n_e and n

$$\begin{gathered} n_e(t) = \frac{|A|}{k_1k_2N}\left[\mathrm{tg}^2\left\{\sqrt{\frac{|A|}{2}}(t - |B_2|)\right\} + 1\right], \\ n(t) = \frac{\gamma N}{k_2} + \frac{\sqrt{2|A|}}{k_2}\,\mathrm{tg}\left\{\sqrt{\frac{|A|}{2}}(t - |B_2|)\right\}, \end{gathered} \tag{7}$$

where $B_2 = \sqrt{\frac{2}{|A|}}\arctan\frac{(k_2n(0) - \gamma N)}{\sqrt{2|A|}} < 0$. It follows from (7) that when $t = |B_2|$, n_e reaches a minimum and then increases without limit, the increase having an explosive character. The time taken for n_e and n to become infinite is $t_i = |B_2| + \pi/\sqrt{2|A|}$. The neglect of the "depletion" of the excited particle in (1) at the instant when n_e increases $n_e(t = |B_2|)$ is justified if $k_1 \gg \gamma$. In addition, the time t_i must be much less than the time of quenching collisions when excited particles collide with gas molecules.

In order that the mechanism considered should in fact lead to explosive development of electrons after the ionization source is switched off, it is necessary to show that after the time for which the source acts the condition $A < 0$ is realized. For this purpose we solved system (1) for $Q \neq 0$ and $n_e(0) = n(0) = 0$. When $t \leq T_1 \equiv \gamma^2N/2k_1k_2Q$ the solution has the form

$$n_e(t) = \frac{2}{k_1k_2N}\left[-\frac{1}{4T_2^2}\left(1 - \frac{t}{T_1}\right) + F^2(t)\right],\qquad n(t) = \frac{\gamma N}{k_2} + \frac{2}{k_2}F(t). \tag{8}$$

where

$$F(t) = \frac{1}{2T_2}\left(1 - \frac{t}{T_1}\right)^{1/2} \frac{(I_{-2/3}(z) - DI_{2/3}(z))}{(I_{1/3}(z) - DI_{-1/3}(z))}; \ T_2 = \frac{1}{\gamma N};$$

$$z(t) = \frac{1}{3}\frac{T_1}{T_2}\left(1 - \frac{t}{T_1}\right)^{3/2}; \ D = \frac{I_{1/3}(z_0) + I_{-2/3}(z_0)}{I_{-1/3}(z_0) + I_{2/3}(z_0)} > 0, \ z_0 = z(0);$$

and $I_\nu(z)$ is the cylindrical function of imaginary argument. Equation (8) generalizes the quasistationary solution given in [2], obtained assuming that $dn_e/dt = 0$ and holding in the region $T_2 \ll t \ll T_1$ for arbitrary ratios between T_1 and T_2. Assuming the functions $I_{\pm 2/3}$ and $I_{\pm 1/3}$ to be positive and taking into account the nature of their variation in the region $0 \le z \le z_0$, it can be shown that the solutions (8) for n_e and n have singularities. If $t \ge T_1$, we have for n_e and n

$$n_e(t) = \frac{2}{k_1 k_2 N}\left[-\frac{1}{4T_2^2}\left(\frac{t}{T_1} - 1\right) + \Phi^2(t)\right], \quad n(t) = \frac{\gamma N}{k_2} - \frac{2}{k_2}\Phi(t), \tag{9}$$

where

$$\Phi(t) = \frac{1}{2T_2}\left(\frac{t}{T_1} - 1\right)^{1/2} \frac{(J_{-2/3}(y) - DJ_{2/3}(y))}{(J_{1/3}(y) + DJ_{-1/3}(y))},$$

$$y = \frac{1}{3}\frac{T_1}{T_2}\left(\frac{t}{T_1} - 1\right)^{3/2},$$

and $J_\nu(y)$ is the cylindrical function of the first kind. The solution (9) has a singularity at the point defined by the following equation:

$$J_{1/3}(y) + DJ_{-1/3}(y) = 0, \tag{10}$$

where the constant D lies in the interval $1 \le D < \infty$ when the parameters T_1 and T_2 vary. Using tabulated values of the functions $J_{1/3}$ and $J_{-1/3}$, it can be shown that for any values of D in this interval the least root y_1 of Eq. (10) lies in the limits $1.8 \le y_1 \le 2.4$. Putting $y_1 \simeq 2$ approximately, we obtain an estimate for the time t_i' taken for the instability to develop when $Q \ne 0 (t_i' - T_1) \simeq 3.3\, T_2^{2/3}\, T_1^{1/3}$. If $T_2 \ll T_1$, we have $t'_i \approx T_1$, which is practically identical with the results obtained in [2]. When $T_2 \gg T_1$ we have $t'_i \approx 3.3\, T_1^{1/3}\, T_2^{2/3} \gg T_1$.

We will use the solutions (8) and (9) to analyze whether the condition $A < 0$ is satisfied at the stage when the ionization source acts. If we introduce the function $A(t) \equiv (1/2)\,[nk_2 - \gamma N]^2 - k_1 k_2 N n_e$, it is seen from (8) and (9) that at an arbitrary instant of time it is given by the expression

$$A(t) = \frac{1}{2T_2^2}\left(1 - \frac{t}{T_1}\right). \tag{11}$$

As follows from Eq. (11), $A(t) \ge 0$ when $t \le T_1$ and $A(t) < 0$ when $t > T_1$. This means that if the duration of the ionization source T is less than the characteristic time T_1 and the quantity $A(t = T)$, which determines the constant A in (5), is positive, after switching off the ionization source the concentration of the electrons and excited particles approaches zero. To obtain an unlimited increase in n_e and n after switching off the ionization, it is necessary for the duration of the ionizing source to exceed the time T_1.

Since the delay time of the instability t_i after switching off the source is of the order $[A(t = T)]^{-1/2}$, taking (11) into account, it is defined by the ratio between the times T, T_1, and T_2. In particular, if $T = T_1 + \Delta T$, where $\Delta T \ll T_1$, we have $t_i \sim T_2(T_1/\Delta T)^{1/2} \gg T_2$.

The author thanks I. V. Tyutin for participating in a discussion and for analyzing the solutions obtained.

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ORIGINATION OF A SELF-OSCILLATING MODE (MAGNETIC STRIATIONS) IN A NONEQUILIBRIUM MAGNETIZED PLASMA

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UDC 533.951

In this paper, quantitative computations of the nonlinear solution of the problem of ionization instability development in a bounded domain [1], performed by the Lyapunov–Schmidt method [2], are presented. The amplitude of the self-oscillations is computed, the domains of the hard and soft modes of the loss of stability are isolated, a distribution of the electron density and electrical current over the channel section is constructed for the soft mode of the loss of stability – nonlinear magnetic striations. The topology of the striations in the post-critical domain is discussed. It is shown that the maximum of the steady-state wave amplitudes does not correspond to that wave which first lost stability. The results obtained are used for a qualitative analysis of experimental results with a nonequilibrium magnetized plasma in a magnetic field (the existence of oscillations at small wavelengths in a full ionization mode of the admixture).

§1. Let us examine the behavior of a nonequilibrium magnetized plasma in a domain bounded by two nonconducting walls x = 0 and x = b, which are infinite in the y direction. The magnetic field induction vector is directed along the z axis. Let us assume the parameters of heavy particles (atoms and ions) to be independent of the coordinates and time, while the ionization equilibrium build-up time is considerably less than the characteristic time of the problem. We consider the Reynolds magnetic number small and we neglect the effects of radiation. Taking account of these assumptions, the system of equations describing the state of the medium reduces to a dimensionless system of n partial differential equations in the potential Φ_n and the electron concentration Θ_n [1]. The system is solved by the method of a series expansion in the small supercriticality parameter $\varepsilon = (\Omega - \Omega^-)/\Omega^{-1}$. In a zero approximation (n = 0) the system has the form

$$L_{11}^0\Phi_0 + L_{12}^0\Theta_0 = 0,\ L_{21}^0\Phi_0 + L_{22}^0\Theta_0 = 0 \tag{1.1}$$

with the boundary conditions (see [3])

$$\Phi_0(0,\ Y) = \Phi_0(1,\ Y) = 0,\ \Theta_0(0,\ Y) = \Phi_0(1,\ Y) = 0, \tag{1.2}$$

where

$$L_{11}^0 = \frac{d^2}{dx^2} - k_y^2;\ L_{12}^0 = -a_1\frac{d}{dx} - ik_y\Omega^-;$$

$$L_{21}^0 = 2\frac{d}{dx};\ L_{22}^0 = -\Lambda L_{11}^0 + f_1;\ Y = y + W_0t;$$

Λ is a small parameter; Ω^- is the critical Hall parameter; k is the wave vector; and a_1 and f_1 are constant factors [1, 4].

The solution of (1.1) with the boundary conditions (1.2) can be represented in the form

Irkutsk./Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 22-30, May-June, 1978. Original article submitted April 28, 1977.

 0021-8944/78/1903-0296$07.50